2021 ASSESSMENT RESOURCE

A-level
MATHS

Differentiation and Integration (Topics G,H)

Version 1.0

Total number of marks: 40

$*1)\ \int (6x^2)^{-1}\, dx = \frac{1}{6}\int x^{-2}\, dx$

$= \frac{1}{6}x - x^{-1} + c$

$= -\frac{1}{6x} + c$

1 Given that $\frac{dy}{dx} = \frac{1}{6x^2}$ find y.

Circle your answer.

[1 mark]

$\frac{-1}{3x^3}+c$ $\qquad$ $\frac{1}{2x^3}+c$ $\qquad$ $\frac{-1}{6x}+c$ $\qquad$ $\frac{-1}{3x}+c$

3 It is given that

$$y = 3x^4 + \frac{2}{x} - \frac{x}{4} + 1$$

Find an expression for $\frac{d^2y}{dx^2}$

[3 marks]

$y = 3x^4 + 2x^{-1} - \frac{1}{4}x + 1$

$\frac{dy}{dx} = 12x^3 - 2x^{-2} - \frac{1}{4}$

$\frac{d^2y}{dx^2} = 36x^2 + 4x^{-3}$

5 Differentiate from first principles

$$y = 4x^2 + x$$

[4 marks]

Let $f(x) = 4x^2 + x$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{4(x+h)^2 + x + h - 4x^2 - x}{h}$

$= \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 + x + h - 4x^2 - x}{h}$

$= \lim_{h\to 0} \frac{8xh + h^2 + h}{h} = \lim_{h\to 0} 8x + h + 1$

As $h \to 0$ then

$f'(x) = 8x + 0 + 1$

$\therefore f'(x) = 8x + 1$

5 $f'(x) = \left(2x - \frac{3}{x}\right)^2$ and $f(3) = 2$

Find $f(x)$.

[4 marks]

$f'(x) = (2x - 3/x)^2$

$= 4x^2 - 12 + 9x^{-2}$

$f(x) = \int f'(x)\,dx = \frac{4}{3}x^3 - 12x - 9x^{-1} + C$

$f(3) = \frac{4}{3}(27) - 36 - \frac{9}{3} + C = 2$

$= -3 + C = 2$

$\Rightarrow C = 5$

$f(x) = \frac{4}{3}x^3 - 12x - 9x^{-1} + 5$

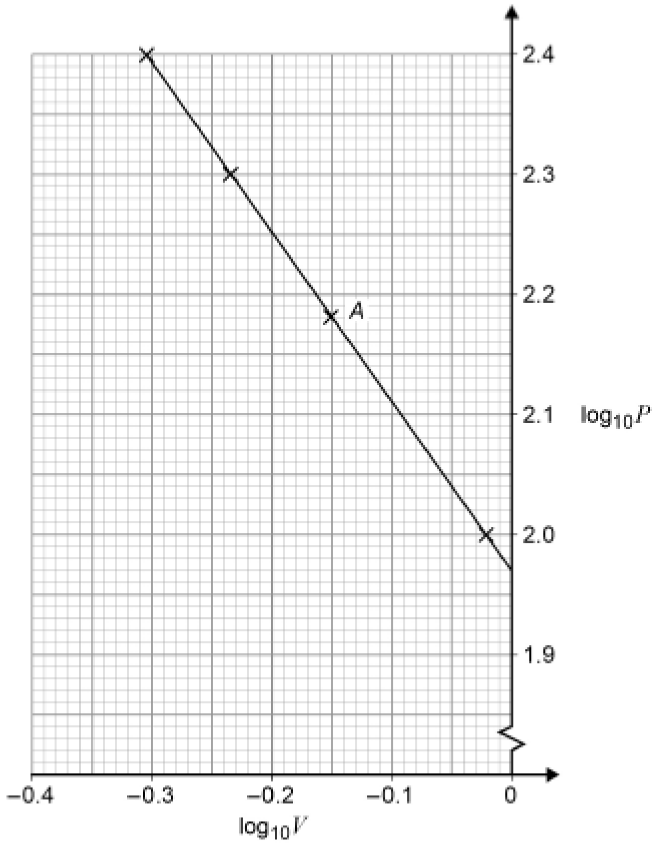
8 Maxine measures the pressure, P kilopascals, and the volume, V litres, in a fixed quantity of gas.

Maxine believes that the pressure and volume are connected by the equation

$$P = cV^d$$

where c and d are constants.

Using four experimental results, Maxine plots $\log_{10}P$ against $\log_{10}V$, as shown in the graph below.



8 (a) Find the value of P and the value of V for the data point labelled A on the graph.

[2 marks]

(a) $P = 10^{2.18} = 151$

$V = 10^{-0.15} = 0.71$

8 (b) Calculate the value of each of the constants c and d.

[4 marks]

(b) $(0, 1.97)$

$10^{1.97} = c$

$P = cV^{d}$

$151 = 10^{1.97}(0.71)^{d}$

$d \ln 0.71 = \ln \frac{151}{10^{1.97}}$

$d = \frac{1}{\ln 0.71} \times \ln\left(\frac{151}{10^{1.97}}\right) = -1.4$

9 (a) (i) Find

$$\int (4x - x^3)\,dx$$

[2 marks]

(a)(i) $\int (4x - x^3)\,dx = 2x^2 - \frac{1}{4}x^4 + C$

9 (a) (ii) Evaluate

$$\int_{-2}^{2} (4x - x^3)\,dx$$

[1 mark]

(ii) $\int_{-2}^{2} (4x - x^3)\,dx = \left[2x^2 - \frac{1}{4}x^4\right]_{-2}^{2}$

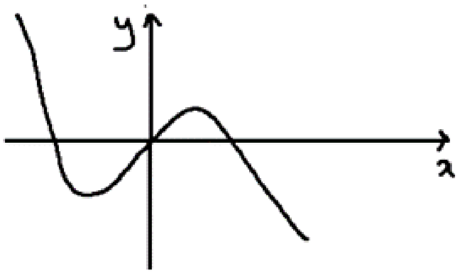
$= 2(2)^2 - \frac{1}{4}(2)^4 - 2(-2)^2 + \frac{1}{4}(-2)^4$

$= -4 + 4 = 0$

9 (b) Using a sketch, explain why the integral in part **(a)(ii)** does **not** give the area enclosed between the curve $y = 4x - x^3$ and the x-axis.

[2 marks]

(b)



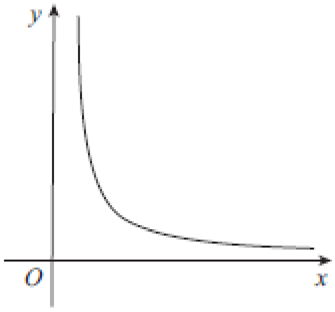
As the graph is symmetric about $x=0$, the integral from -2 to 0 of the integral is negative the integral from 0 to 2 so they cancel out and don't give the area

9 (c) Find the area enclosed between the curve $y = 4x - x^3$ and the x-axis.

[2 marks]

(c) $2\left|\int_{-2}^{0} 4x - x^3 \, dx\right| = 2(4) = 8$

6 A curve has equation $y = \dfrac{2}{x\sqrt{x}}$



The region enclosed between the curve, the x-axis and the lines $x = 1$ and $x = a$ has area 3 units.

Given that $a > 1$, find the value of a.

Fully justify your answer.

[5 marks]

$$y = \frac{2}{x\sqrt{x}} = 2x^{-1}(x^{-\frac{1}{2}})$$

$$= 2x^{-3/2}$$

$$\int_1^a 2x^{-3/2}\,dx = \left[-2(2)x^{-\frac{1}{2}}\right]_1^a = \left[-4x^{-\frac{1}{2}}\right]_1^a = -4a^{-\frac{1}{2}} + 4$$

$$-4a^{-\frac{1}{2}} + 4 = 3$$

$$-a^{-\frac{1}{2}} + 1 = \frac{3}{4}$$

$$-a^{-\frac{1}{2}} = -\frac{1}{4}$$

$$\Rightarrow a = 16$$

8 A curve has equation

$$y = x^3 + px^2 + qx - 45$$

The curve passes through point R (2, 3)

The gradient of the curve at R is 8

8 (a) Find the value of p and the value of q.

[5 marks]

a) $y\big|_{x=2} = 8 + 4p + 2q - 45 = 3$

$\Rightarrow 4p + 2q = 40$ ①

$\frac{dy}{dx} = 3x^2 + 2px + q$

$\frac{dy}{dx}\Big|_{x=2} = 3(2)^2 + 4p + q = 8$

$\Rightarrow 4p + q = -4$ ②

① − ② $\Rightarrow q = 44$

Sub into ① $4p + 2(44) = 40$

$\Rightarrow p = 12$

8 (b) Calculate the area enclosed between the normal to the curve at R and the coordinate axes.

[5 marks]

b) gradient of normal at $x = 2$ is $-\frac{1}{8}$

$y - y_1 = -\frac{1}{8}(x - x_1)$

$y - 3 = -\frac{1}{8}(x - 2)$

$y = -\frac{x}{8} + \frac{13}{4}$

So $(0, \frac{13}{4})$ and $(26, 0)$

Area $= \frac{bh}{2} = \frac{1}{2}\left(\frac{13}{4}\right)(26) = \frac{169}{4}$